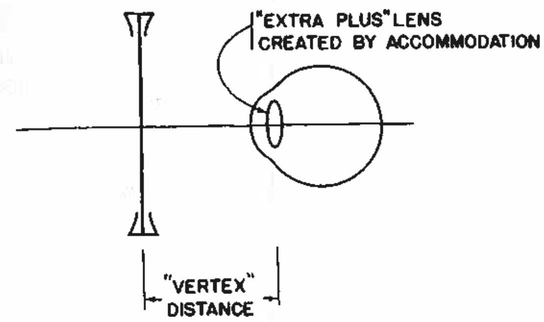
Since, in both instances, the eye is myopic 8.7 D (referred to the cornea) it is relieved of the responsibility of accommodating to the extent of the first 8.7 D. So, with the − 10 D *corrective* lens, this eye must accommodate (12.25 − 8.7) or 3.55 D to see the object located 20 cm from the spectacle plane; whereas the eye corrected by a *contact* lens must accommodate (13.35 − 8.7) or 4.65 D — that is, 1.10 D more!

(Notice from this example we assume that the contact lens-corrected myope must exert the same amount of accommodation as the emmetrope would. However, even *with* a contact lens, the myope will likely have to exert somewhat *less*. This discrepancy arises because, in our model eye, we neglected the distance behind the cornea that the myopic eye error truly "resides"; because of this oversimplification then, we find that the contact lens corrected eye is equivalent to the emmetropic one.)

CLINICAL POINT:

You are performing the finishing touches on a subjective refraction of a patient. If, after you arrive at his full correction, you still push on and continue to add minus lens power, the patient will tell you that the target letters seem to become smaller. Why?

The *extra* minus power you have added stimulates the patient's accommodation. That *increase* in accommodation can be looked upon as an increase in the plus power "built-in" his eye. These two dioptric powers (minus outside, plus inside) *must* "neutralize" each other exactly if vision is to remain clear; and so, we have created an additional "neutralized" or afocal telescopic system with the two elements separated by the existing *vertex* distance (plus some *more* distance within the eye to the site of the built-in plus). Since here, the "built-in" lens (which corresponds to the telescope's "eyepiece") is plus, this Galilean system is a *minifying* one for this eye.



So, placing extra minus in front of an eye forces a patient to peer through a small, reverse telescope. The greater in minus the over-correction is, the greater the accommodation required to keep the letters clear, and therefore the more the minifying effect of that telescope.

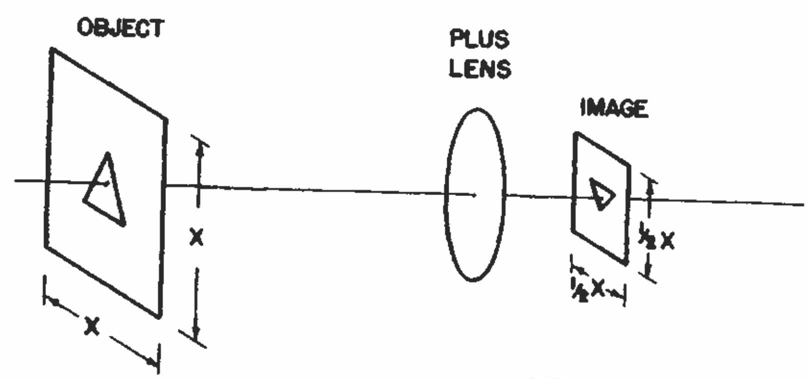
MAGNIFICATION

So far, we have glibly tossed around the various *types* of magnification. We have mentioned *linear* (lateral) magnification in connection with basic lens imagery and introduced *angular* magnification when dealing with Galilean and astronomical telescopes. We have even implied the presence of a third type — called *axial*. All are often confusing, so now I want to describe each type separately and discuss it briefly so you could get the proper "feel" for all of them.

Linear Magnification

Linear magnification has to do only with the sizes of images relative to their corresponding objects — not with how big images *look* to an eye, but how big they *are*. The image formed by a lens system can be larger, smaller or the same size, and it can be erect or inverted when compared to the original object. As long as we deal only in the relationships between the actual sizes (meters, inches, or microns) of objects and images, *linear* magnification is the term that applies.

If we know the linear magnification is $\frac{1}{2}$ X, we know the image is half as long and half as wide as the object (the measurements taking place in the planes perpendicular to the axis of the optical system).



LINEAR (LATERAL) MAGNIFICATION

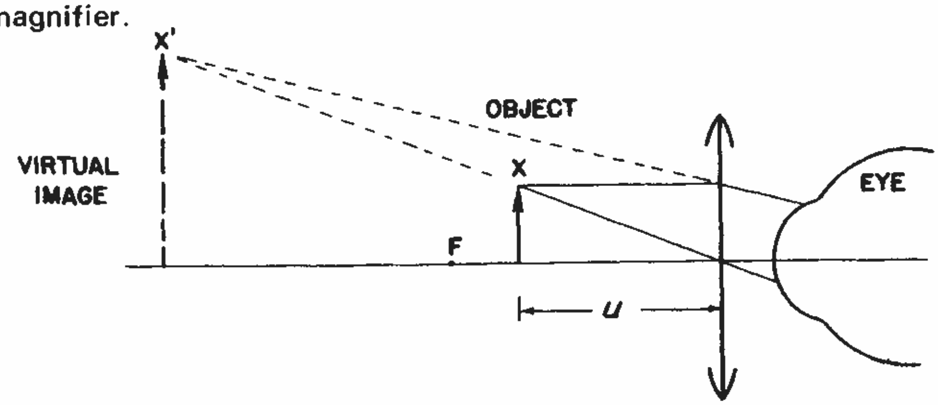
But *linear* magnification has no real meaning when dealing with objects at *infinite* distances. Since the linear magnification = $\frac{\text{image size}}{\text{object size}} = \frac{\text{image distance}}{\text{object distance}}$, if the *object* distance is infinite, the linear magnification must be zero. When the *image* distance is infinite (as when an object is placed in the focal plane of a simple plus lens), it makes no difference what the *actual* object size is, the image's size (since it is located at infinity) must be infinitely large and, therefore, the *linear* magnification is infinitely large also. In both these instances, the term linear (also called *transverse* as well as *lateral*) magnification has no usefulness.

In optics and ophthalmology we need a way to deal *not* with absolute sizes, but with apparent sizes; how big or small an object or image *looks* to the eye. Angular size and angular magnification provide us with this means.

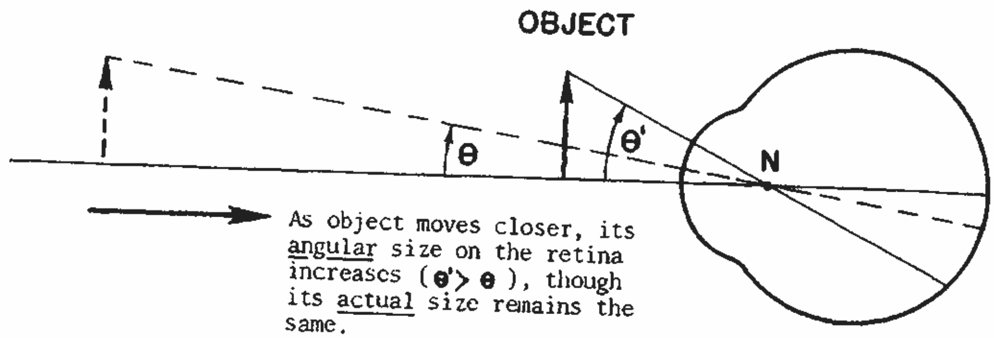
Angular Magnification

Angular size has to do with how big an image *looks* to an eye. The same fixed size of object or image will appear larger if you approach it and smaller as you recede. The image on your retina is magnified angularly (that is, it subtends a larger angle at the eye's nodal point) as an object is brought closer. But, it is only *magnified* angularly in relation to some other angular size — that given by that same object at some fixed reference distance. For example, say an object located at an arbitrary reference distance (say 20 ft.) subtends 5° on my retina; when it approaches to half its previous distance (10 ft.), its angular size on my retina has increased to 10° — a 2 X angular magnification has occurred. If instead, the object recedes to twice its original distance (to 40 ft.), its angular size on my retina has shrunk to 2.5°, and the angular magnification has become only ½ X. But this is only with reference to its angular size at the 20 ft. distance. So, it should be obvious that in order to state a magnification figure, you *must* give (or assume) a reference distance; the *change* in appearance from that reference angular size *is* the angular magnification provided by any optical device. This point is so important and yet it is often misunderstood.

When the object is at *infinity* and an angularly magnified image is supplied by a telescope, there is really no need to specify any other fixed reference distance to calculate the angular magnification since the angular size of the *object* at infinity can serve as *the* reference. However, for magnifying objects close-by, we *do* need a reference distance, as you will see. Let's examine how a simple plus lens works as a magnifier.

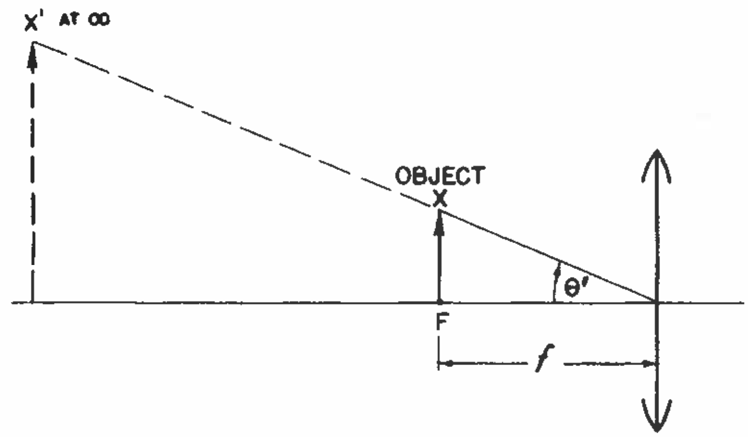


Whenever any object is placed just *inside* the primary focal point of a simple plus lens, a magnified virtual, upright image will be formed. An eye situated behind that lens will see the image. If we fix the object distance *u* from the lens, the image size and position will also be fixed. Any eye movement relative to the lens cannot influence the *actual* image size; however, its apparent (angular) size *may* change, just as moving closer or farther from any real life object would change its retinal image size. (As the object in the figure below moves closer to the eye, its *angular* size increases on the retina ($\theta' > \theta$) though its *actual* size stays the same.)

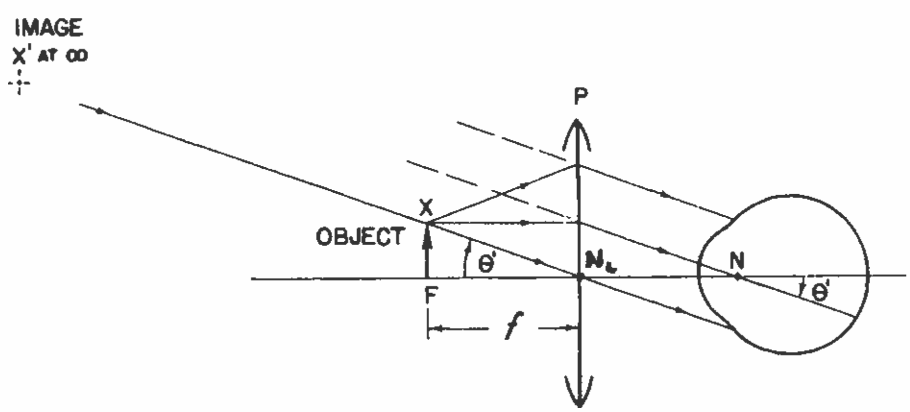


I want particularly to emphasize the word *may* (influence the apparent size) because it *may not* also! Say the image is located at a great distance away, that is, if object distance *u* is equal to the focal length of the simple plus lens; now if the eye moves closer or farther from the lens (mm or yards), the *angular* size of that image on the retina will *not change.* It would only change if your eye moved an *appreciable* distance away from the lens as compared to the image distance, just as, for example, if you were five miles away from a mountain and moved a few meters closer to it; this would not really increase the visual size of that mountain. To accomplish that, you'd have to move say ¼ to ½ mile closer. Only then might you notice any increase in its angular size on your retina. Similarly here, the image is at infinity and you would not observe any shrinkage in its angular size as you receded from it while keeping it in view through the lens.

As shown in the diagram below, θ' is the angular size of the *object* subtended at the lens; θ' is also always the angular size of the *image* subtended at the lens.



When the object is at F of the plus lens magnifier, the angular size of the *retinal* image created will also be θ'. How do we know this?



Well, you should be able to see that *all* rays from the object tip X will, in the image space of the lens, be parallel and must all cross the axis at the same angle as that ray which goes through the *lens'* nodal point;

that angle is θ'. Since *one* of those image rays will pass through the *eye's* nodal point, the angular size of the retinal image will be θ' too! And, it will always be θ' unless the eye is so far back that light from X which passes through lens P cannot enter the pupil — a situation which depends on the *diameter* of the lens and not its power.

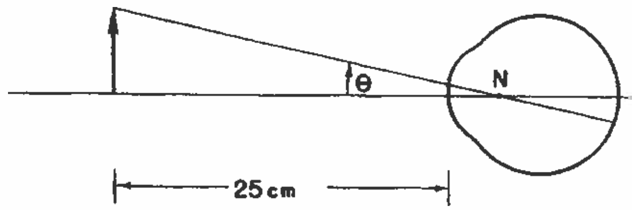
The angular size of θ' does depend on the size of the object and on the focal length f_1 of the lens. Study the last diagram; as f_1 gets shorter, the retinal image size θ' must get larger; that is, θ' is inversely proportional to f, or stated more clearly, the greater the dioptric power P of the lens, the greater θ' will be. Remember that we are still considering only small angles, so tan $\theta' = \theta'$; therefore,

$$\theta' = \frac{XF}{f} = \frac{\text{object size}}{f} = (\text{object size}) \cdot (\text{Power of the lens}).$$

But to determine how much "magnification" lens P offers us, we must be able to compare *this* retinal image size θ' with some *other* one having to do with the same object. Remember, magnification is a relative term which states that a retinal image is bigger or smaller than *something*. So our task is to find a reference distance at which to place that same object, determine its angular size in that instance, and compare that angular size with the θ' created by lens P. You *could* put that object anywhere you wanted and obtain some magnification "number". *Where* to put that object is what must be agreed upon (not calculated, but arbitrarily picked) as a standard.

What that standard distance should be has been argued about for years, but for most purposes 25 cm has been agreed upon and is used here and in most other textbooks. However, you should know that there are certain situations where 40 cm is a better standard. The specific letter sizes of Louise Sloan's reduced-vision acuity charts, for example, are calibrated in M (magnification) units for a 40 cm reading distance. At that position, a patient with poor acuity is asked to read. If a given sized print can just barely be read, the M unit label for that sized print "tells" an examiner directly the power of a low-vision aid which might be a useful one to try first for that patient.

But as stated, the *standard* reference distance is 25 cm. An object on the visual axis would subtend angle θ when it was placed 25 cm from an eye. (See next figure.)



We can return now to the angular magnification provided by lens P — the simple plus lens magnifier. With small angles:

$$\theta' = (\text{object size}) \cdot (\text{Power of the lens})$$

and

$$\theta = \frac{\text{object size}}{25 \text{ cm}} = \frac{\text{object size}}{.25 \text{ meters}}$$

$$M = \frac{\theta'}{\theta} = \frac{\text{object size} \cdot (P)}{\frac{\text{object size}}{.25}} = \left(\frac{1}{4}\right) P$$

and the magnification is seen to be independent of what the object size actually is.

The ¼ represents our chosen standard distance; if *another* distance *d* is used as a reference, the magnification then would be $P \times d$ (in meters).

So we have found that the angular magnification of a simple plus lens magnifier $= \frac{P}{4}$; so, a + 8 D sphere is called a 2 X magnifier, and a + 20 D sphere is called a 5 X magnifier.

The same reasoning applies to the eye itself. If an eye, *without* any magnifiers, looks at any object located at our reference distance of 25 cm, it also would obtain a *unitary* (1 X) angular magnification. If that same object is now brought closer, to 12.5 cm, and if that eye can accommodate the 8 D required to see it clearly, the angular size on the retina will be doubled and the angular magnification due to this proximity and to the required accommodation will become 2 X. Thus, it is clear that an increase in the accommodative response of an eye can add to the magnification supplied by a simple plus magnifier.

With a 25 cm reference distance, the magnification given by the eye's accommodation is in a ratio of $\frac{4 \text{ Diopters}}{1 \text{ Mag unit}}$, the same as that provided by a lens; so, the magnification due to accommodation *adds directly* to that due to the lens itself. To determine the *full* magnification possible with a simple lens, we use the *general* expression which takes into account the eye's accommodative ability; that expression is $\frac{P + A}{4}$. In our previous example, then, a + 8 D (2 X magnifier) lens, when coupled with an eye which has, say, 8 D of accommodative amplitude, will provide a total possible magnifying effect of $\frac{8 + 8}{4}$ or 4 X.

CLINICAL POINT:

The important clinical implication of the above is that a 10 year old child with poor (say 20/100) vision at 20 feet will rarely require much in the way of a high plus simple magnifier (as that given by a high bifocal add) for reading. Accommodative amplitude is usually ample to allow him to bring reading material close enough to provide a sufficiently enlarged retinal image without requiring any further help from a lens. Kestenbaum pointed out that a good approximation of the near add required by a patient with reduced vision is the reciprocal of his visual acuity at distance. An acuity of $\frac{20}{100}$ would require $\frac{100}{20}$ or a + 5 D add to aid in near work.* For the child in the above example, with sufficient accommodative power, this add would probably be superfluous, as explained.

* * *

Previously we showed that moving the eye back from the plus lens magnifier will *not* change the angular magnification power of the lens and will not change the image size on the retina (assuming the object is in the anterior focal plane of the magnifier). But something does change as the eye moves back, and that is the field of view. The extent of the image which is visible to the eye is maximal when the eye is in contact with the lens but decreases as the eye recedes.

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* That is, to read that size of print which is about the size of pica typewriter type — the 14/42 letter size on the 14" near-acuity card.

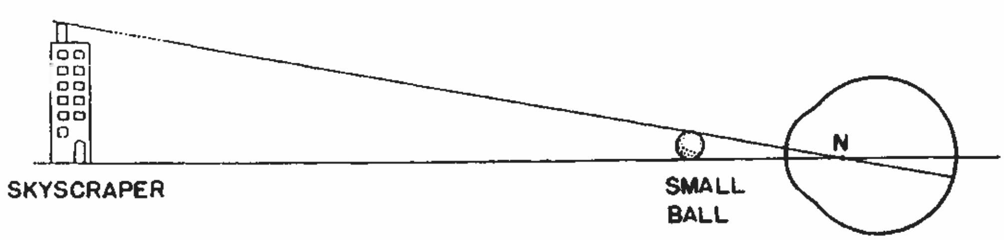
CLINICAL POINT:

The direct ophthalmoscope, our handy-dandy gadget for examining the interiors of the eye, makes use of the eye's dioptric components as a simple magnifier. The ophthalmoscope itself is only a flashlight, arranged so that you can sight just alongside the light beam. All those lenses on its dial are only to compensate for your refractive error, the patient's refractive error, any amount of accommodation you exert (but shouldn't) and the height or depth of any fundus lesion you are examining. It is very likely all these will sum to zero and you will need no supplementary lens in your ophthalmoscope dial to see the fundus clearly. Then, you are using the patient's cornea and lens powers combined as a plus magnifier.

We learned that the dioptric power of an emmetropic eye is roughly 60 D. Thus the angular magnification of fundus detail is $\frac{P}{4} = \frac{60}{4} = 15$ X; that is, the optic nerve head (which is in reality about 1.5 mm in diameter) appears 15 times larger through the ophthalmoscope than it would look if it were situated 25 cm from your eye with the lens and cornea removed.

* * *

Our psychological apparatus is such that we are enabled to substitute a change in an object's angular *size* for a change in its *distance* from ourselves, and, we can do so rather freely, but within limits. Thus, when the image size of an object which *can* exist in various sizes is increased on the retina, we can perceive either that the object itself has enlarged or that it came closer to us; usually, we "feel" the latter has occurred. But, if we *know* that its distance couldn't change, the increase in angular size will be interpreted as the object enlarging.



As shown in the above diagram, the angular image height on the retina of both the ball and the skyscraper is the same. Since a ball can come in any size, its retinal image size does not psychologically fix its distance away and the ball may be "seen" either as a small one up close or a large one farther away. A building, however, is *known* to be of a certain size, therefore, it is the *distance* that becomes the variable.

With both the skyscraper and the ball, I am assuming that no other cues are present which help set the distance more accurately — such as shadow, color, overlap, etc.* Any of these cues (as well as your "mental set") can tip the scales towards your making a clear choice as to whether it was distance or size that *really* changed. Typically, telescopes and binoculars seem to bring objects "closer" since the objects visualized are familiar ones whose size is known. Microscopes on the other hand make objects appear "bigger" since there is no such familiarity with the object's true size. All these instruments, of course, can only enlarge the size of the retinal image of the object.

So, angular magnification (in contrast to linear magnification) involves the eye or a camera, and the true size of the image matters only indirectly, since it is only *one* of the variables, the other being its *distance.* The term "angular magnification" takes both variables into account in one datum which hides the *actual* value of either. Your personal judgment of the apparent magnitude of either will depend on the balance of many psychological factors, but for our subject "angular magnification", it is only the *angular size* that counts.

Axial Magnification

There is yet another type of magnification which spills over into both the linear and angular types; this is *axial* magnification. It is not usually discussed in the standard optics text. I am mentioning it here so that you can be aware of its presence and of its particular significance, especially in relation to indirect ophthalmoscopy.

You have been shown that with a telescope the stress on an observer's accommodation is proportional to the *square* of the *angular* magnification provided by the instrument. This is a manifestation of

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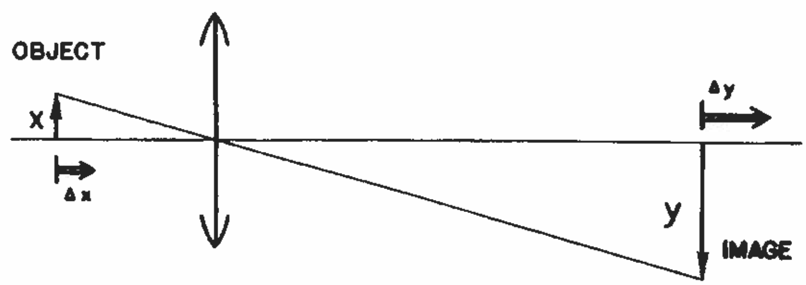
*Rubin and Walls: *FUNDAMENTALS OF VISUAL SCIENCE, pp. 359-403.*

how a small change in the object's axial (fore and aft) distance (that is, as it comes closer) causes a *magnified* axial shift in the position of the final image; thus the final image appears to be *much* closer and thereby demands much more accommodation. *This* is axial magnification.

Even when dealing with *linear* magnification (in the figure below) $(M = \frac{\text{image size}}{\text{object size}} = \frac{y}{x})$, *axial* shifts ($\Delta x$) of the object will create *axial* shifts (Δy) in the image.

AXIAL MAGNIFICATION



These shifts $(\frac{\text{axial image shift}}{\text{axial object shift}})$ are proportional to $(\frac{y}{x})^2$, that is, to the square of the linear magnification. (This is detailed in one of my previous articles and will be summarized later in the section on the indirect ophthalmoscope, page 292.) So, the *axial* magnification is related to the square of the *linear* magnification* just as it is to the square of the *angular* magnification.

Now you shouldn't be caught unaware when you see the term *axial* magnification again.

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* This is not quite true when one deals with a refracting *surface*, which, of course, has different indices of refraction for the object and image spaces. For the refracting surface, the axial magnification $= (\frac{n}{n'} \times \frac{v}{u}) \times \frac{v}{u}$. (Call this relationship "Formula One.") If you will recall, I showed (in Appendix A) that, with a refractive surface, the linear (lateral) magnification M was equal to $(\frac{n}{n'} \times \frac{v}{u})$. So, by substitution in "Formula One," $M_{axial} = (M_{linear}) \times \frac{v}{u}$. But, with a lens in air, $M_{linear} = \frac{v}{u}$. So, for the *lens*, $M_{axial} = (M_{linear}) \times (M_{linear}) = (M_{linear})^2$. This is what is stated in the text above.